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COMMENT

Comment on 'A single-sum expression for the overlap integral of two associated Legendre polynomials'

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Abstract

The overlap integral of two associated Legendre polynomials (ALPs) presented by Mavromatis (Mavromatis H A 1999 *J. Phys. A: Math. Gen.* **32** 2601) is revised. The results are valid for any product of two ALPs with arbitrary degree l and order m.

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The overlap integral of $P_{l_1}^{m_1}(x)$ and $P_{l_2}^{m_2}(x)$ in the interval [-1, 1] is well known only for the special case $m_1 = m_2$ [1]. However, we cannot find the overlap integral of two ALPs for the general case $l_1 \neq l_2$ and $m_1 \neq m_2$, namely,

$$I(l_1, m_1; l_2, m_2) = \int_{-1}^{1} P_{l_1}^{m_1}(x) P_{l_2}^{m_2}(x)\,\mathrm{d}x. \tag{1}$$

Recently a closed expression of the overlap integral of two ALPs involving a single sum has been derived from equations (5) and (6) given in [2]. The final result given in (7) of [2], however, is *invalid* for arbitrary m_1 and m_2. When $(-m_1 + m_2)$ is negative, the phase $(-1)^{m_1}$ involved in $C(l_1, m_1; l_2, m_2)$ of equation (7) in [2] should be changed to $(-1)^{m_2}$. This is a consequence of the restriction $m \geqslant 0$ required by equation (5) of [2]. Hence the use of equation (5) of [2] will be *invalid* for the *negative* $(-m_1+m_2)$. The coefficients a_k are evaluated explicitly in [3], where

$$P_n^{\alpha,\beta}(x) P_m^{\gamma,\delta}(x) = \sum_{k=0}^{m+n} a_k P_k^{\lambda,\mu}(x). \tag{2}$$

The formula given in [2] is the special case $\lambda = \mu = 0$, $\alpha = \beta$ and $\gamma = \delta$.

For completeness we write out the *correct* result as

$$I(l_1, m_1; l_2, m_2) = A(l_1, m_1; l_2, m_2) \sum_l D(|m_2 - m_1|, l)(2l + 1) \times \begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l \\ -m_1 & m_2 & m_1 - m_2 \end{pmatrix}, \tag{3}$$

where

$$A(l_1, m_1; l_2, m_2) = (-1)^\tau \; |m_2 - m_1| 2^{|m_2 - m_1| - 2} \sqrt{\frac{(l_1 + m_1)!(l_2 + m_2)!}{(l_1 - m_1)!(l_2 - m_2)!}}, \tag{4}$$

$D(|m_2 - m_1|, l)$ is given in (7) of [2] and τ takes the following values:

$$\tau = \begin{cases} m_1, & \text{if} \quad m_2 \geqslant m_1, \\ m_2, & \text{if} \quad m_2 < m_1. \end{cases} \tag{5}$$

This result differs from equation (7) of [2] for the phase factor. The two cases ofτ were not considered in [2]. This result can also be understood from the symmetry $I(l_1, m_1; l_2, m_2) = I(l_2, m_2; l_1, m_1)$.

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[2] Mavromatis H A 1999 *J. Phys. A: Math. Gen.* **32** 2601, and references therein
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